Pilot Ignition of Cold Supersonic Flows

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An experimental and theoretical study is reported of the ignition of a cold, ducted supersonic flow of premixed ethylene and air by means of a coaxial pilot flame. The boundary between conditions leading to ignition and conditions leading to quenching of the flame has been investigated experimentally, and compared with predictions from a computer model of the turbulent combustion process. Comparison has also been made with a simple ignition analysis based on a ratio of characteristic chemical and mixing length scales. Satisfactory agreement has been found between the experimental data and results of both theoretical studies.

Nomenclature

C_p = specific heat
D = pilot diameter
D_{crit} = minimum D to obtain ignition
E = activation energy
F = pre-exponential factor [Eq. (2)]
l = distance to initiate reaction
l_m = minimum value of l
L = potential core length
m_{ox} = oxidizer mass fraction
m_{fu} = fuel mass fraction
p = static pressure
R = gas constant
T = static temperature
T_0 = total temperature
U = velocity
x = axial distance from nozzle exit
ϕ = stoichiometric ratio, fuel-to-air
β = mole fraction of combustible
τ_i = ignition delay time
τ_h = heat release time
τ = characteristic reaction time

Subscripts
m = minimum value
1 = conditions in cold combustible flow
2 = conditions in hot jet

I. Introduction

PILOT flames are used to stabilize combustion in many propulsion systems. For example, use of a pilot flame has been suggested[1] in order to extend the operation of a supersonic combustion ramjet to low-flight Mach numbers where spontaneous ignition of the fuel-air mixture will not occur. References 2–6 are among the many studies of pilot-flame ignition in the literature.

Ignition by a pilot flame or other heated jet occurs as the hot gas mixes with and heats the combustible gas. However, ignition will not occur if the hot jet is too small, because then it will be too quickly cooled by mixing with the surrounding gas, and the heated region will be too small to allow the chemical reactions to build up a propagating flame. Combustion is then quenched by too rapid mixing between the hot and cold streams. This paper is concerned with the conditions which must be met in order to ensure that ignition will occur. Since the pilot flame must often be placed in the fuel-air mixing region, it must ignite a highly turbulent, imperfectly mixed, high-speed flow.

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Quantitative predictions of turbulent combustion processes are difficult to make. Turbulent mixing and the chemical kinetics of combustion reactions are imperfectly understood at the present time so that, even in isolation, the description of each must be largely empirical. When turbulence and combustion occur together the possibility arises of coupling between them, with the rate of reaction influenced by the turbulence and the rate of mixing influenced by the reaction. Although this coupling has long been recognized as a possibility, very little evidence is available as to its magnitude and even the most complex computer models of turbulent combustion generally ignore it completely. However, it must be accepted that present uncertainties in both reaction rate and mixing rate models for practical systems are very large, and that coupling between turbulence and mixing will be detected only if its influence is sufficient to produce an effect greater than the sum of these uncertainties.

The ignition of a cold, combustible stream by a hot, coaxial turbulent jet is not only a practical problem relevant to the design of combustion chambers, it is also an important fundamental problem. This is because the coaxial geometry is the simplest arrangement for which theory and experiment can be compared, and because the ignition region from a pilot jet is a region where chemical and turbulent mixing length scales are comparable, where coupling between turbulence and reaction might be expected. A comparison of predictions from turbulent combustion models with experiments on pilot-flame ignition and quenching provides a sensitive test of the validity of the models.

The present work consists of an experimental and theoretical study of the ignition of a cold, premixed, supersonic flow of ethylene and air by means of a hot turbulent pilot jet. The experiments have determined the location of the boundary between ignition and nonignition of the fuel-air stream by the hot jet. A computer model of this flow has been developed and has been used to make predictions of the ignition/nonignition boundary for comparison with the experimental data. This model makes use of an empirical gradient-dependent description of turbulent transport which appears to work well in unreacting flows. It also uses a global reaction rate equation giving a good fit to experimental data on the ignition of ethylene and air by a low-speed laminar jet. In view of the many empiricisms and uncertainties necessarily incorporated into this relatively sophisticated computer model, a much simpler ignition criterion has also been tested both against the computer predictions and against the experimental data. The criterion relates the boundary between ignition and nonignition to a ratio of characteristic chemical and mixing lengths in the flow.

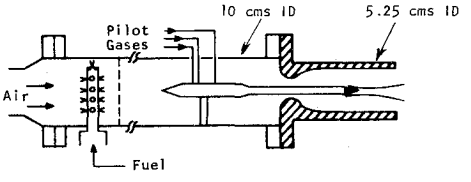

Fig. 1 Experimental apparatus.

This paper contains a preliminary report of a continuing program of research into turbulent combustion. Pilot-flame ignition studies have also been carried out in nonuniform flows and in imperfectly mixed fuel-air streams. The results of these studies have been reported separately.[7]

II. Experimental Study of Ignition by Means of a Pilot Flame Apparatus

Figure 1 shows a sketch of apparatus used in this study. Fuel and preheated air were mixed in a settling chamber before being accelerated to supersonic velocity through an axisymmetric nozzle. The flow then passed through a pyrex glass test section of constant cross-sectional area but variable length, before exhausting to atmosphere. The pilot-flame device was placed on the axis of symmetry of the nozzle and test section. Major dimensions are shown in Fig. 1.

Effectiveness of the mixing chamber was tested by sampling in the nozzle exit plane. Ethylene and air were mixed in the chamber and samples collected in the flow were analyzed with a gas chromatograph to verify that mixing was complete.

The nozzle, exit diameter 5.25 cm, was designed for an exit Mach number of 2.0 by Foelsch's[8] method. The method was modified slightly to allow for the presence of the pilot burner tube cantilevered through the center of the nozzle and also for the boundary layer existing on the nozzle and pilot tube walls. A simple area transformation was made at each axial station computed, such that the nozzle contour was enlarged to give a free area equal to that calculated when these blockage effects are absent. The nozzle performance was checked by determining the total pressure levels at many radii in the exit plane and found to be satisfactory. The measured Mach number at the exit plane varied over a range within ± 0.02 for all points at a distance greater than 0.15 cm from the walls.

Figure 2 shows a detailed drawing of the pilot burner assembled in the mixing chamber. The burner is a modified version of that described in Ref. 4. Hydrogen and air were premixed, hydrogen-rich with an equivalence ratio of approximately three, and combusted at the burner tip with just sufficient oxygen being added to react with the excess hydrogen. Ignition was achieved by use of a spark plug located in the hydrogen-air stream and energized with a Ford Model T ignition coil system, which produces continuous sparking at the plug as long as it is energized. The plug was only used to ignite the pilot, however, and once ignited, it was de-energized.

Prior to installation in the mixing chamber, the pilot was tested in the laboratory. The mass flow rates of the three gases fed to the pilot were controlled by the use of calibrated capillary tubing and the temperature beneath the spark plug was monitored with a thermocouple. Experiments were made to establish an operating mode in which combustion would be limited to the burner tip, allowing the pilot body to remain cold. Such an operating condition could be achieved, it was discovered, if the pilot was ignited with a mixture equivalence ratio of approximately four. Lower ratios tended to give rise to stable combustion near the spark plug well causing the pilot body to increase in temperature very rapidly. Readjusting the mass flows during combustion to the original design values did not affect the mode of combustion. There was found to be a lower limit of mass flow for which stable tip-combustion could be achieved; hydrogen flow rates greater than 0.44 kg/hr were always sufficient to give good performance. The maximum flow rate tested, limited only by the hydrogen pressure regulator, was 1.54 kg/hr. This was considered to be sufficient for testing purposes. Since the composition ratio of the pilot gases was held constant, changes in the pilot flow rate should not influence the temperature of the pilot jet, assuming complete combustion within the pilot tube. The calculated adiabatic flame temperature for all test conditions was 2648°K. The length of the pilot tube downstream of the nozzle exit was variable between 5.08 and 45.7 cm in order to study the influence of the thickness of the pilot-tube boundary layer on the ignition process.

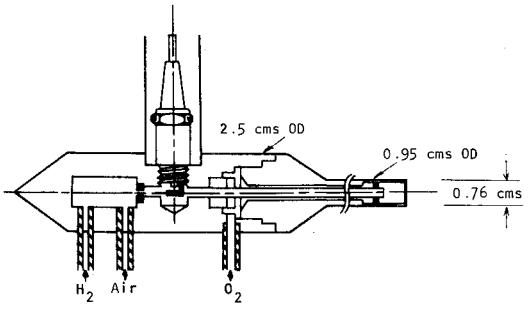

Fig. 2 Details of pilot.

Table 1 Range of test conditions

Nozzle exit static pressure	1 atm
Total temperature of air	375–555°K
Fuel-to-air equivalence ratio	0.4–1.0
Nozzle exit Mach number	2.0
Pilot exit temperature	2648°K
Pilot exit velocity	89–310 m/sec
Pilot power level	3–10 kw
Pilot flame diameter	0.76 cm

The apparatus was designed to operate continuously using either hydrogen or ethylene fuel within the range of test conditions shown in Table 1.

The exit static pressure was chosen for convenience, while the maximum total temperature of the air stream was limited by the auto-ignition temperatures of the fuels (approximately 750°K for stoichiometric ethylene-air mixtures). The nozzle size and exit Mach number were decided from consideration of airflow requirements and fuel costs.

Experimental Results

Mixtures of ethylene and air were accelerated to supersonic velocity around the pilot burner into a constant diameter pyrex glass test section. Measurements included Pitot pressure traverses, wall static pressure distributions, and color photographs of propagating flames in the supersonic flow. Radial Mach number profiles were deduced from the Pitot and static pressure measurements upstream of the pilot flame and typical results are shown in Fig. 3. In all tests, the duct length beyond the pilot tip was limited to 6.5 cm in order to prevent thermal choking and subsequent flashback to the mixing chamber.

The flame photographs, which were all taken from a fixed position with constant exposure time, aperture and film sensitivity, were interpreted as showing either ignition or nonignition of the premixed stream by the pilot flame. This interpretation was empirical. It was observed that the luminosity recorded on the film either ceased well within the 6.5 cm between the pilot tip and the end of the test section, or

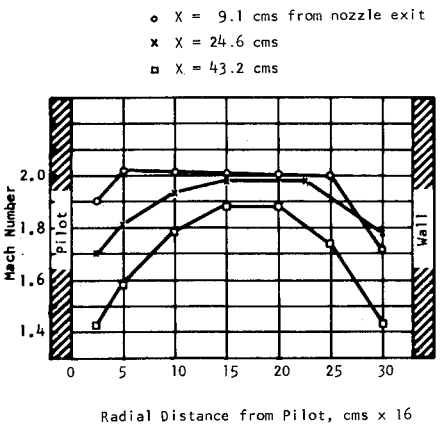

Fig. 3 Variation in Mach number profile with axial position.

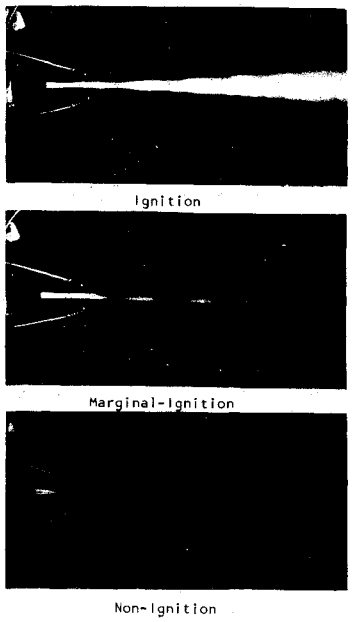

Fig. 4 Typical photographs of pilot flame response.

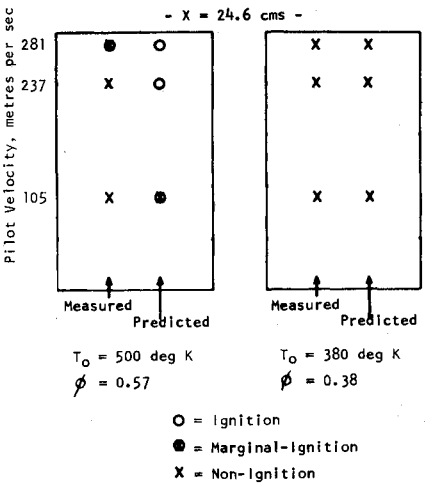

Fig. 6 Effect of pilot velocity upon ignition boundary. Comparison between experimental data and predictions from the computer model.

extended far downstream into the freejet emerging from the test section. Tests giving the former type of photograph were designated as "nonignition" and those giving the latter type as "ignition." A further subdivision was made to separate those ignition photographs which showed an increase in flame diameter with distance from the exit of the test section, from those that did not grow in diameter; for these latter cases, "marginal ignition" was said to have occurred. Typical photographs of these three conditions are shown in Fig. 4.

The ignition/nonignition boundary was investigated in this way for air stagnation temperatures between 380°K and 555°K, fuel-to-air equivalence ratios ϕ between 0.4 and 1.0, at pilot-flame locations 9.1 cm, 24.6 cm, and 43.2 cm downstream of the nozzle exit. In all cases the static pressure was 1 atm at the pilot tip. The pilot-gas exit velocity was maintained constant at 105 m/sec and the results summarized in Fig. 5a were obtained. The effect of pilot flow rate upon the ignition boundary was studied at one axial station with the pilot flame 24.6 cm downstream of the nozzle exit. Two different sets of flow conditions were investigated: air stagnation temperature 555°K, equivalence ratio $\phi = 0.57$; air stagnation temperature 380°K, equivalence ratio $\phi = 0.38$. In each case, tests were made at pilot-flow velocities of 105, 237, and 281 m/sec. The results are summarized in Fig. 6a.

Summary of Trends Observed in Ignition Experiments

1) The strongest effect upon the boundary between ignition and nonignition, clearly shown in Fig. 5a, is that of the fuel-to-air equivalence ratio ϕ. For the lean mixtures tested, a reduction in ϕ made ignition harder to achieve, as would be expected.

2) In most cases, the stagnation temperature of the cold stream was not observed to influence the ignition boundary. However, two cases may be seen in Fig. 5a where a reduction in stagnation temperature at constant ϕ led to quenching.

3) Again in most cases, the length of the pilot tube was not observed to influence the boundary. There is one exception in Fig. 5a, where increasing the pilot tube length is seen to cause quenching. This trend is opposite to that expected from laminar, boundary-layer studies.[9]

4) Figure 6a demonstrates that in most cases the ignition process was found to be insensitive to the exit velocity of the pilot jet at nominally constant pilot temperature. However, one case may be observed where an increase in pilot velocity favored ignition.

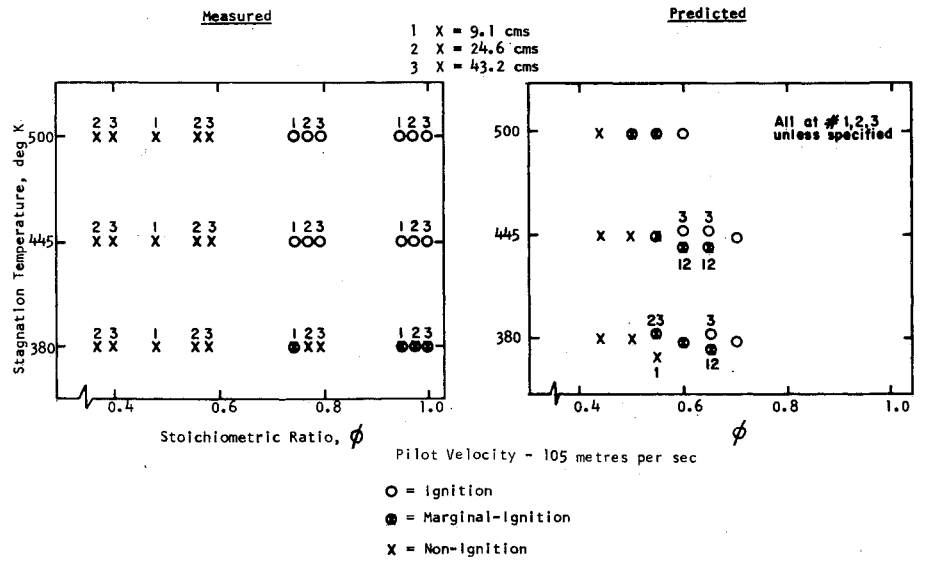

Fig. 5 Effect of stoichiometric ratio, stagnation temperature and axial position upon ignition. Comparison between experimental data (a) and predictions (b) from the computer model.

III. Computer Model of Turbulent Combustion

Description of Model

Only a brief description will be given in the present paper of this general purpose computer program‡ for the solution of problems involving turbulent mixing and turbulent combustion. Details of the program have been published elsewhere.[7] Three basic assumptions are incorporated, concerning the form of the flow equations, the empirical description of turbulent transport, and the rate of the combustion reactions, respectively. These assumptions will be introduced in turn and then the method of solution of the equations will be briefly described.

The first assumption is that the conservation equations for time-averaged flow properties may be approximated by boundary-layer equations, implying that gradients perpendicular to the flow direction are much greater than gradients along the flow, and that pressure variations across the flow are small. With this assumption the equations of conservation of species, momentum, and energy all reduce to the common form

$$\partial\psi/\partial x+(a+bw)\,\partial\psi/\partial w=(\partial/\partial w)[c_\psi(\partial\psi/\partial w)]+d_\psi \qquad (1)$$

where ψ represents the axial flow velocity, the stagnation enthalpy or a species mass fraction, w is a nondimensional stream function, a and b are measures of the mass flow rates across the boundaries of the mixing region, c_ψ is a local transport coefficient, and d_ψ is the source term for quantity ψ.

The second assumption in the formulation of the model is that turbulent transport processes may be approximated by Spalding's modification[10] of the classical mixing length hypothesis, which takes account of freestream turbulence. The mixing length is set equal to a constant across free shear layers and the outer parts of wall shear layers, and this constant is proportional to the local width of the shear layers. Close to walls, the mixing length grows in proportion to the distance from the wall. Turbulent Prandtl and Schmidt numbers are assumed constant throughout the flow. Empirical constants appearing in this formulation are not regarded as quantities to be adjusted from one problem to another, in order to obtain the best possible agreement with experimental data. Information regarding the choice of these constants may be found elsewhere.[7]

Finally, the rate of consumption of fuel due to the combustion reactions is assumed to be given by a simple global expression of the Arrhenius type, namely

$$d_{fu}=-(1/\rho u)Fp^2m_{fu}m_{ox}\exp(-E/RT) \qquad (2)$$

where d_{fu} is the source term in Eq. (1) for the fuel mass fraction, m_{fu}, F is the pre-exponential factor, E the activation energy of the reaction, R the universal gas constant, p the pressure, ρ the density, and m_{ox} the oxidizer mass fraction. Both F and E are assumed to be known constants.

The method of solution of the problem just formulated is a development of that described by Spalding and Patankar.[11] The flow is divided into shear layers and uniform flow regions. Having taken a step downstream in the axial direction x, and assumed a value for the pressure gradient dp/dx, the finite-difference form of Eq. (1) is solved step-by-step across each shear layer. The thickness of each shear layer is then determined from the rates of entrainment at its boundaries. Quasi-one-dimensional flow equations applied to the uniform flow regions then allow the width of these regions also to be calculated. Because the pressure gradient dp/dx was assumed rather than calculated, the total width of the flow will generally not coincide exactly with the given boundaries. Rather than setting up a time-consuming iteration loop, dp/dx is corrected at the next step. With suitable controls the deviation of the computed flow boundary from the specified wall shape is negligibly small. A result of this and other features is that the program is rapid, efficient, and economical in use.

‡ Developed by D. B. Spalding of Imperial College, London, England, Consultant to Northern Research and Engineering Corporation.

Kinetics of Ethylene Combustion

The mechanism and reaction rates for the combustion of ethylene are not well understood at the present time. White[12] has measured the ignition delay time of very lean mixtures of C_2H_4 in O_2, while Mullaney et al.[13] have measured the ignition delay time in fuel-rich mixtures of C_2H_4 and air. Also, Fenimore and Jones[14] have studied the rate of consumption of hydrocarbon in low-pressure, premixed $C_2H_4/H_2/O_2$ flames. Their expression for the rate of removal of C_2H_4 requires knowledge of the concentrations of H, OH, and O. The kinetic expression of Eq. (2) is clearly empirical. There is little justification for its assumption that the reaction rate is first-order with respect to both fuel and oxidizer; Mullaney et al.[13] found that the ignition delay time was first-order in oxygen concentration for fuel-rich mixtures, but White[12] found an over-all first-order dependence for very lean mixtures. No clear guide could be found from the behavior of other hydrocarbons.

The empirical approach used here is based on the experiments of Vanpee and Bruszak.[2] These authors studied the ignition of stagnant mixtures of air and combustible gases by means of a small, low speed, laminar jet of inert gas, whose temperature was increased until ignition occurred. Goh and Ma[3,15] have made predictions which are in striking agreement with this experimental data for propane, ethane, and ethylene, using a computer program similar to that described in Sec. III of this paper. Their computation for ethylene used a global rate equation similar to Eq. (2), with

$$F=0.537\ \text{kg m/N}^2; \qquad E/R=19160^\circ\text{K}$$

In view of the success of these calculations, the same values of F and E have been used in the computation of the ignition process in high-speed turbulent flows, as described in the following section.

Computed Results

The computer program has been used to study the boundary between ignition and nonignition under conditions simulating those of the experiments described in Sec. II. Computations were started well upstream of the end of the pilot tip, and the boundary layers on the pilot tube and the outer wall of the duct were properly represented as the initial conditions were set to equal measured values. Figure 7 shows predicted isotherms for a case where the pilot-flow velocity is 350 m/sec, $\phi=0.6$ and $T_0=500^\circ$K.

There was no difficulty in distinguishing between cases where ignition occurred and cases where it did not. When ignition did not occur, the temperature close to the axis of symmetry first

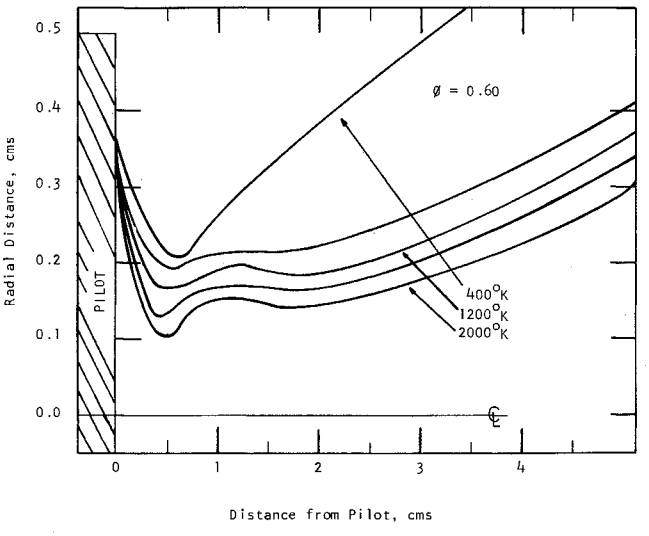

Fig. 7 Computed isotherms.

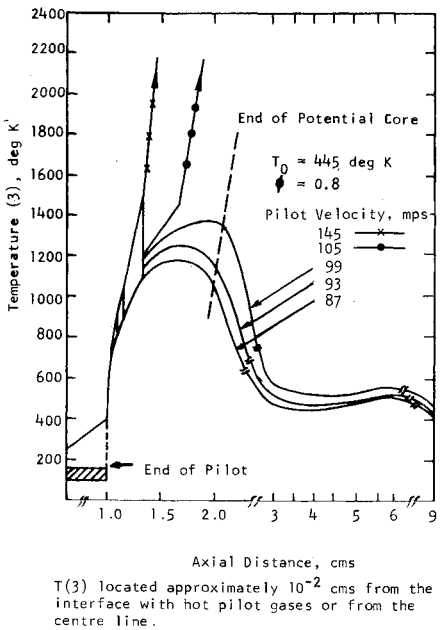

Fig. 8 Temperature profile on a grid line near the inner boundary of the cold combustible flow.

rose (Fig. 8) due to mixing with hot gas from the pilot, but the temperature rapidly fell again at the end of the potential core of the mixing region between the pilot and the outer flow. In these cases, a negligibly small amount of fuel was burnt, either within the potential core or further downstream. When ignition occurred, on the other hand, the temperature rose sharply (Fig. 8) before the end of the potential core, and fuel was consumed within the potential core. It was noted that, where ignition was "marginal", as defined in Sec. II, the first signs of ignition (temperature rise, fuel consumption) always occurred very close to the end of the potential core. This is the basis of the simple ignition criterion developed in Sec. IV.

Figures 5b and 6 summarize the results of the machine computations. Agreement with the experimental data can be observed to be generally good. At a fixed exit velocity of 105 m/sec, the dominant parameter affecting ignition is predicted to be the fuel-to-air equivalence ratio ϕ. Cold-stream stagnation temperature is shown to influence the ignition boundary at slightly lower pilot-exit velocities when $\phi = 0.8$. Contrary to experimental measurement, however, ignition is shown to be made easier by an increase in pilot length.

IV. Simple Ignition Criterion

Origin of Criterion

Any computer model of turbulent combustion must be severely limited in accuracy at the present time, because of large uncertainties concerning the rates of turbulent mixing and chemical reaction. Because these uncertainties can lead to errors many times greater than any computational errors, it might be expected that a very much simpler model could yield equally useful results. For this reason, an attempt has been made to derive a simple criterion to predict the boundary between ignition and nonignition.

The starting point of the analysis is the work of Marble and Adamson,[16] who solved the problem of the two-dimensional, laminar mixing of two parallel, semi-infinite streams, one cold and combustible and the other hot and inert. As a result of mixing between the streams, the combustible gas is heated and, because of the infinite extent of both streams, ignition always occurs. The reaction proceeds slowly near the boundary between the mixing region and the cold combustible gas, because the temperature there is low. It also proceeds slowly very close to the boundary between the hot, inert gas and the mixing region, because the concentration of combustible gas is small. Between these limits lies a path where the reaction rate is greatest and hence where the reaction length is a minimum. The calculations show that the temperature begins to rise, and a flame begins to form, at the transverse location of this path of maximum reaction rate, after a distance in the flow direction approximately equal to the minimum reaction length.

The Marble-Adamson analysis has been extended by various authors.[17] In particular, Tsuji[6] treated the case where the hot gas stream is a two-dimensional laminar jet of finite width. His results clearly show that, if the jet width is less than a critical value, the hot jet is rapidly cooled by mixing with cold gas from the outer flows, and combustion is quenched. An important feature noticeable in Tsuji's calculations is that marginal ignition is obtained when the length required for combustion and heat release to become evident in the temperature profile is similar in magnitude to the length of the inviscid core of the laminar mixing region calculated for the nonreactive case. Shorter reaction lengths lead to combustion whereas longer ones lead to quenching of the flame.

Together with the results of the computer model (Sec. III), these studies suggest the use of an ignition criterion based upon a comparison between the minimum possible length required for chemical reaction and a length characteristic of the dimensions of the mixing zone. The results also indicate that these lengths can be estimated from flow properties in an unreacting mixing region, thus greatly simplifying the calculations. This criterion is developed below for the ignition of a cold, combustible stream by means of a hot, turbulent, coaxial jet of inert gas.

Analysis

Figure 9 defines the conditions to be considered, in which a cold combustible stream of velocity U_1, and temperature T_1 is to be ignited by a hot inert jet of velocity U_2, temperature T_2, and initial diameter D. The following assumptions are made.

1) A characteristic chemical reaction length l may be calculated from time-averaged thermodynamic and flow properties of an unreacting jet, and is not influenced by diffusion of reaction intermediates or products.

2) The turbulent mixing process in the ignition region may be approximated by neglecting effects of chemical reactions and by assuming turbulent Prandtl and Schmidt numbers of unity.

3) The pressure p is assumed constant within the small region where ignition or quenching occurs.

Consider first the near field of the turbulent mixing zone (Fig. 9), in which a uniform core of gas from the hot jet remains intact. In the absence of significant heat release from chemical reactions, the length L of this core is approximately[18]

$$L = 11D\{\rho_2 U_2/\rho_1 U_1\}^{1/2} \tag{3}$$

Because of the assumption that the turbulent Prandtl and Schmidt numbers are unity, the normalized profiles of temperature, velocity, and unreacting composition across the mixing layer will be similar. Thus, in the near field

$$(T - T_1)/(T_2 - T_1) = (U - U_1)/(U_2 - U_1) = 1 - \beta = F(\eta) \tag{4}$$

where T, U, and β are the local temperature, flow velocity, and mole fraction of combustible mixture, at a point within the mixing

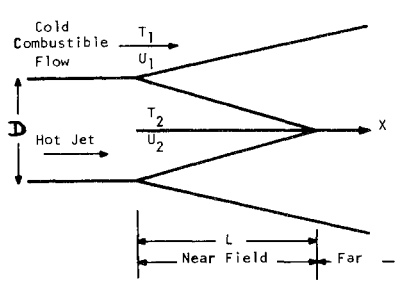

Fig. 9 Idealized mixing model.

zone; $\beta = 1$ at the outer edge of the mixing zone, and $\beta = 0$ at the inner edge. The quantity η is a normalized coordinate in the direction perpendicular to flow. It is assumed that the rate of progress of the combustion reaction may be characterized by a unique reaction time, τ, with a functional dependence

$$\tau = \tau(T, p, \beta, \phi) \tag{5}$$

where ϕ is the fuel/oxidizer mixture ratio, defined to be unity for a stoichiometric mixture. It is further assumed that this function is known. The reaction length is

$$l = U\tau \tag{6}$$

which is a function of η because of Eq. (4). The minimum value of l, l_m, may be found by differentiating Eq. (6) and setting the result equal to zero, giving

$$\tau(\partial U/\partial\eta) + U(\partial\tau/\partial T)\,\partial T/\partial\eta + U(\partial\tau/\partial\beta)\,\partial\beta/\partial\eta = 0 \tag{7}$$

if ϕ and p are constant through the mixing layer. If the functional dependence of τ upon T and β is specified, these equations can easily be solved to obtain the quantities U_m, T_m, and β_m, which are the velocity, temperature, and composition leading to the minimum reaction length l_m.

In the far field of the mixing zone, without reaction, the temperature, velocity, and composition differences will decay approximately as $(L/x)^2$, where x is the distance downstream from the hot jet exit. That is, at a distance $x = 2L$, the maximum temperature difference has fallen to $(T_2 - T_1)/4$. Assuming that T_2 is much greater than T_1, the resulting rapidly falling temperature may be expected to have a dominant effect on the reaction. Consequently, if sufficient combustion to keep the temperature up has not occurred within a distance $x \lesssim L$, the reaction will be quenched. Thus, the core length L is the proper mixing length to compare with the minimum reaction length l_m. Early ignition is to be expected when $l_m/L \ll 1$, ignition will be marginal when $l_m/L = 0(1)$, and quenching of combustion reactions is to be expected when $l_m/L \gg 1$.

Ignition of Ethylene in Air

An appropriate reaction time for the ignition of ethylene/air mixtures may be deduced from Eq. (2). Because of the exponential in this expression, the predicted rate of reaction is controlled mainly by temperature. Quenching will occur, unless the reaction can proceed fast enough to counteract the fall in temperature due to mixing, and so τ must be chosen as the time characterizing the rate of rise of temperature due to combustion, in the absence of effects due to mixing. Thus, τ is defined as

$$\tau = [(1/T)\,dT/dt]_s^{-1} \tag{8}$$

where $(dT/dt)_s$ is the initial rate of rise of temperature due to reaction alone, at given temperature, pressure, and unreacted composition. Using Eq. (2) with the same values of E and F as in the computer model, and also the same specific heats and heats of formation, Eq. (8) becomes

$$\tau\beta^2 \frac{P}{P_{\text{atm}}} = 1.2\times 10^{10} \frac{(1+\phi/14.75)^2}{\phi} \exp(19160/T)\,\text{sec} \tag{9}$$

where P_{atm} is standard atmospheric pressure.

With τ obeying Eq. (9), Eqs. (4 and 7) may easily be solved to give U_m, T_m, and β_m. Assuming, as confirmed numerically, that β_m is much less than unity, it is found that

$$T_m = T_2\left[1 - \frac{2}{E/RT - (1 - U_1/U_2)/(1 - T_1/T_2)}\right] \tag{10}$$

approximately, and U_m and β_m follow immediately from Eq. (4). Equations (6, 9, and 3) may then be combined to give the criterion quantity l_m/L in the following form

$$\frac{l_m}{L} = \frac{3.18\times 10^{-12}}{D}\frac{P_{\text{atm}}}{P}\frac{(1+\phi/14.75)^2}{\phi} \times U_2\left\{\frac{\tau_2 U_1}{T_1 U_2}\right\}^{1/2}\left[\left(1-\frac{T_1}{T_2}\right)\frac{E}{RT_2}\right]^2 \times (1+N)(1+3N)\exp\left\{\frac{E/RT_2}{1-2RT_2/E(1+N)}\right\} \tag{11}$$

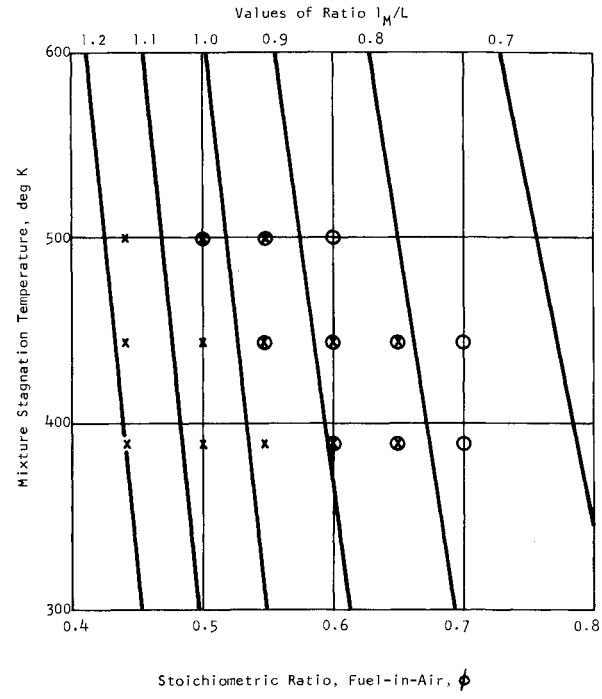

Fig. 10 Comparison between ignition predictions for ethylene-air flows with a pilot flow velocity of 100 m/sec. The straight lines show values of the criterion quantity l_m/L obtained from Eq. (11), while the data points are computer predictions for a pilot tube length of 9.1 cm, reproduced from Fig. 5b. It may be noted that l_m/L is close to unity in the region of marginal ignition.

where D is measured in metres, U_2 in m/sec, E/R is 19,160°K, and

$$N = (U_1/U_2 - 1)/[E/RT_2(1 - T_1/T_2)]$$

It may be seen that l_m/L is a function of D, p, ϕ, U_1, U_2, T_1, and T_2.

Since this criterion makes use of an approximation to the rate equation employed in the computer model, the accuracy of predictions obtained from the criterion may best be assessed by comparison with results from the computer model. Figure 10 shows such a comparison for a range of values of stagnation temperature T_0 and the equivalence ratio ϕ of the ethylene/air flow. It may be seen that, as expected, the criterion quantity l_m/L is close to unity under conditions giving marginal ignition, while smaller values go with successful ignition and larger values with nonignition. The criterion correctly predicts that, for $\phi < 1$, weaker mixtures are less easy to ignite, and it also predicts the slight trend to easier ignition at higher stagnation temperatures. Since it does not include boundary-layer growth, the criterion predicts no influence of pilot length upon the ignition boundary. Similar criterion calculations have yielded results concerning the effects of pilot-flow velocity which are also in agreement with the computer predictions of Fig. 6b. Thus, with $T_0 = 500$°K and $\phi = 0.57$, ignition at a pilot velocity of 105 m/sec is found to be marginal, increasing power makes ignition slightly easier and decreasing power makes it harder.

The simple ignition criterion has therefore reproduced the important trends from both the computer predictions and also the experimental data on ignition and nonignition. Because of the simplicity of the criterion, it can easily be used to explore pilot-flame ignition under a wide range of conditions. As an example of such applications, Fig. 11 shows a prediction of the minimum hot jet diameter, D_{crit}, for successful ignition of a given ethylene/air flow as a function of pressure p, and hot jet temperature T_2, assuming that ignition will occur if $l_m/L \leqslant 1$.

Ignition of Hydrogen in Air

In this case the chemical kinetic mechanism controlling ignition is well documented. The simple model used here assumes

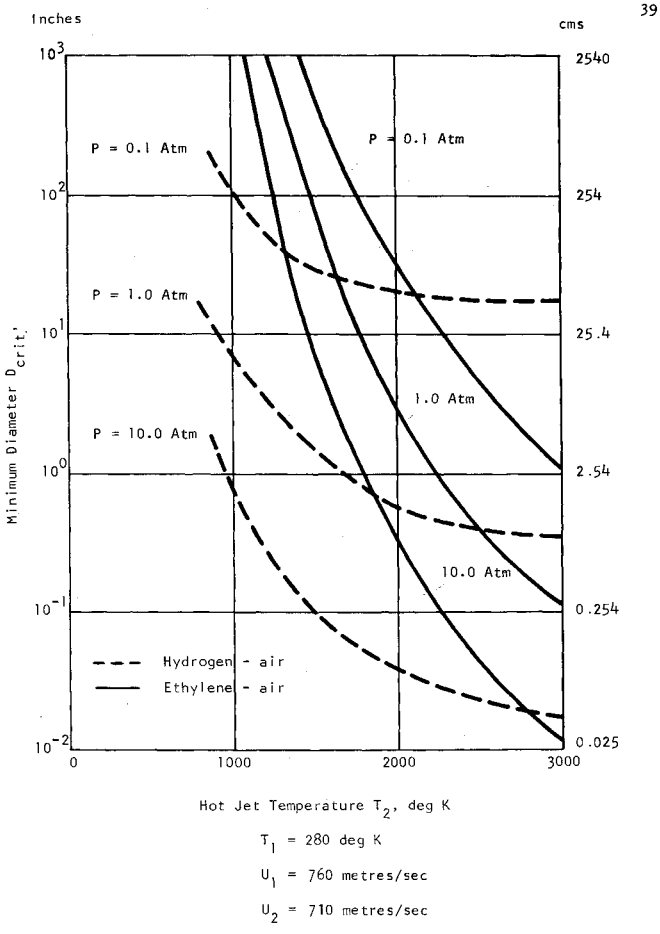

Fig. 11 The minimum jet diameter necessary for ignition to occur. As predicted by the simple ignition criterion for the case $l_m/L = 1$.

τ to be made up of an ignition delay time τ_i and a heat release time τ_h, so that

$$\tau = \tau_i + \tau_h \tag{12}$$

where

$$\beta p \tau_i = G_i f_i(\phi) T \exp(\theta_i/T) \tag{13}$$

and

$$\beta p^2 \tau_h = G_h f_h(\phi) \tag{14}$$

In the preceding equations G_i, G_h, and θ_i are constants, while $f_i(\phi)$ and $f_h(\phi)$ are functions representing the dependence of the characteristic times on the fuel/air equivalence ratio ϕ. The ignition delay time for the combustion of hydrogen in oxygen or air has been studied by many workers. White and Moore[19] have shown that experimental results are fitted over a range of four decades in mixture ratio, by an expression which reduces to Eq. (13) with

$$G_i = 6{\cdot}2 \times 10^{-11} \text{ atm-sec/°K}; \qquad f_i = (1+0{\cdot}42\phi)/(\phi)^{1/2}$$
$$\theta_i = 7150°K$$

On the other hand, there is a lack of reliable information on the heat release time τ_h. This quantity was therefore calculated from the rate of removal of radicals H, OH, and O by the dominant three-body recombination reactions, using the definition

$$\tau_h = \{-(1/R)\, d[R]/dT\}_s^{-1}$$

where s denotes initial conditions and $[R]$ represents the total mole concentration of all radicals, which form a common pool because of fast binary reactions. Because $d[R]/dt$ is second order in $[R]$ for three-body recombination, τ_h is inversely proportional to $[R]$ and thus inversely proportional to β as shown in Eq. (14). The calculations, which assumed a partial equilibrium among the fast binary reactions, suggested that the magnitude of τ_h, as well as the magnitude and direction of its variation with T and ϕ, are all strongly affected by the nature of the third bodies in the three-body recombination reactions. Since the most numerous third body will vary from point to point in the flow, no simple expression can be expected to give a proper representation of τ_h under all circumstances. For the present purpose, a mean value of τ_h was adopted with

$$G_h = 10^{-5} \text{ atm}^2\text{-sec}; \qquad f_h = 1$$

in Eq. (14). These values are appropriate to temperatures of the order of 2000°K and mixture ratios close to stoichiometric.

With τ specified, the criterion quantity l_m/L may be calculated in a manner similar to that described above for the case of ethylene and air. The minimum pilot-jet diameter D_{crit} has again been calculated as a function of pressure and pilot jet temperature, and the results are shown as the dashed curves in Fig. 11. It may be seen that, unlike the ethylene/air case, D_{crit} for the ignition of hydrogen/air mixtures is not inversely proportional to pressure.

V. Discussion

In general, the agreement obtained between the experimental data, computer predictions, and results from the simple model is considered to be satisfactory. Many explanations can be put forward for the small discrepancies between the various sets of results.

Considering the experiments first, the flow in the important region around the tip of the pilot tube may be more complex than has been assumed in the analysis. The process of balancing the static pressures between the subsonic pilot jet and the supersonic outer flow requires an expansion or contraction of the pilot stream by an amount depending on both the pilot flow rate and the length of the pilot tube. Also, if the pilot flow rate is too small, a large recirculation zone will form[20] on the end of the pilot tube, and the resulting increase in the dwell time of the combustible mixture must be expected to favor early ignition. Experiments by Winterfeld[21] have demonstrated the effectiveness of a recirculating flow flame holder in supersonic combustion. However, experiments[20] show that a recirculation zone will not form if the total pressure ratio of the pilot stream to the outer flow is greater than 0.12, whereas, the present experiments employed values of 0.27 or more, indicating that a recirculating flow should not occur. Finally, combustion of pilot gases occurs inside the end of the pilot tube, heating the extreme end of its tip. Heat transfer from the tip of the pilot tube to the cold combustible mixture may be an important factor in ignition which has been neglected in the analysis. This may help to explain why the influence of pilot tube length as determined experimentally is opposite to both the computer prediction and to that expected from the adiabatic wall boundary-layer analysis of Cheng and Kovitz.[9]

It must be clear from Figs. 5a and 6 that more data points are required in order to obtain accurate experimental information on the location of the ignition/nonignition boundary. Unfortunately, the scope of the experimental program was curtailed because of other demands on the test plant.

Both the kinetic model and the values of its empirical constants used to represent the combustion process are, unfortunately, always suspect. The hot gases from the pilot flame in the experiments contain atoms and radicals which may assist, for example, in the ignition of the main flow, whereas the calculations take no account of this effect. A rigorous treatment of the ignition and combustion of ethylene in air requires a kinetic mechanism containing many elementary reactions. The computation technique described here can incorporate a complex, coupled kinetic scheme, but the mechanism and rates are not known. Similarly, turbulent mixing rates are subject to uncertainties, particularly in the presence of large temperature and density gradients, and boundary layers of thickness comparable to the pilot jet width. In this connection, it is encouraging to note that the potential core length predicted from the computer program agrees well with the core length from Eq. (3), and also with the

"luminosity core" visible in some of the color photographs of propagating flames.

The simple ignition criterion has been shown to predict successfully the major factors influencing pilot flame ignition of supersonic ethylene/air flows. This success is interpreted as evidence that the model of ignition and quenching upon which the criterion is based is essentially correct. Some uncertainty is to be expected as to the magnitudes of the characteristic mixing and chemical lengths which occur in the criterion, leading to uncertainty in the exact position of the ignition-quenching boundary. However, the criterion provides a very simple means of predicting trends.

In view of the uncertainties just discussed, the agreement between the experimental data and both theoretical models is considered to be satisfactory. The only disagreement concerns the effect of the pilot tube length upon the ignition boundary; both the observed small adverse effect of increased length in the experiments, and also the predicted small favorable effect in the computer model, probably lie within error bars representing the effects of these uncertainties.

VI. Conclusions

1) Quantitative agreement is obtained between the experimental ignition boundary and predictions of the computer model. This is a sensitive test of the validity of the model.

2) The computer model also confirms the basic postulate of the ignition criterion, that the ignition boundary is related to the ratio of a chemical length to the potential core length of the mixing region between the pilot and outer streams. The criterion provides a simple and apparently successful description of ignition and quenching phenomena.

3) Experiment, computer model, and ignition criterion all agree that increasing the equivalence ratio (lean mixtures) and increasing the pilot-gas flow rate both make ignition easier to achieve.

4) Experiments show that increasing the pilot tube length makes ignition more difficult to achieve, which contradicts the predictions of the computer model.

5) Experiments with long pilot tubes (length 24.6 cm or more) show that increasing the stagnation temperature of the combustible, premixed gas makes ignition easier to achieve. Both theoretical models confirm this trend.

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